A New Mathematical Model for Predicting Waterflood Performance In Oil Reservoirs

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ABSTRACT

In this paper, analytical model is developed for predicting waterflood performance in a stratified reservoir without crossflow between its adjacent layers by assuming saturation gradient exist behind the flood front and there is presence of initial gas saturation at the start of the flood with no trapped gas behind the flood front. The model accounts for variation in porosity, saturation, relative permeability, thickness, height in addition to permeability of different layers. The corner stone of this model is the derivation of models describing position of displacement front in different layers, two-phase injection rate using concept of total mobility and two-phase mobility ratio, two-phase vertical sweep efficiency and modification of single layer area sweep efficiency to suit multilayer reservoir application. The model predicts water injection rate, cumulative oil recovery, cumulative water injected, oil and water production rate, cumulative water produced and water oil ratio at breakthrough and beyond for a stratified reservoir. Validation of this model was done using 17 years injection and production history of a waterflood real field operation, the model result was also compared with most commonly used existing analytical model existing in oil and gas industry. It was observed that the new model closely fit with the field production and injection performance in comparison with existing model that over predicts the waterflood performance. Based on this observation, it can be conclusively said that, a new method of predicting waterflood performance that closely fit with the field performance has been developed, which is also valid for accurate prediction of waterflood performance in either homogenous or stratified reservoirs and can also be used for performance prediction of Water Injection during Pressure Maintenance Operations of Niger-delta oil fields.

Keywords: Vertical Sweep Efficiency, Crossflow, Area Sweep Efficiency, Stratified Reservoir, Pressure Maintenance

1.0 INTRODUCTION

The discovery of crude oil by Edwin L. Drake at Titusville, PA, on Aug. 27, 1859, marked the beginning of Petroleum era. Although the First well produced about 10 B/D (1.6 m^3/d), within 2 years other wells were drilled that flowed thousand barrels per day (Dickey 1959). Production rate from these shallow Pennsylvania reservoirs declined rapidly as the reservoir energy was depleted. Recovery was a small percentage of the amount of oil estimated to be initially in place. As early as 1880, Carll (1880) raised the possibility that oil recovery might be increased by the injection of water into the reservoir to displace the oil to the production wells. The practice of waterflooding apparently began accidentally. The experience in the Bradford field, PA, is typical (Fettke 1938). Many well were abandoned in the Bradford field following the flush production period of the 1880's. Some were abandoned by pulling casing without plugging, while in other wells the casing was left in the wells, where it corroded. In both cases, fresh water from shallower horizon apparently entered the producing interval. Water injection began as early as 1890, when operators realized that water entering the productive formation was stimulating production. By 1907, the practice of water injection had an appreciable impact on oil production from Bradford field.

Waterflooding can be defined as the process of injecting water into the reservoir usually to boost the reservoir pressure and to sweep (displace) oil from the reservoir and push the reservoir fluid towards the production well and thereby increase the production rate and eventually, the ultimate oil recovery**.** The principal reason for waterflooding an oil reservoir is to increase the oil-production rate and, ultimately, the oil recovery. This is accomplished by "voidage replacement"—injection of water to increase the reservoir pressure to its initial level and maintain it near that pressure. The water displaces oil from the pore spaces, but the efficiency of such displacement depends on many factors (e.g., oil viscosity and rock characteristics). Waterflooding also called secondary oil recovery because the process yielded a second batch of oil after a field was depleted by primary production spread slowly throughout the oil- producing provinces. High point of interest in water flooding was developed in the late 1940's and early

1950's when most world reservoirs approached economic limits and operators sort to increase reserves. By 1955, waterflooding was estimated to contribute more than 750, 000 B/D [119,200m3/d] in the U.S. Waterflooding is practiced extensively throughout the world due to water availability and its low cost. In the U.S. alone, as much as half of the current production is taught to be the result of water injection.

Different analytical models are available in the literature for predicting waterflood performance of stratified reservoirs. Stiles (1949) assumed the displacement velocity in a layer to be proportional to its absolute permeability neglecting the effect of mobility ratio. Dykstra and Parsons (1984) developed a model for noncommunicating layers without crossflow between layers while Hiatt (1958) presented a model for communicating layers with complete crossflow. Warren and Cosgrove (1964) applied Hiatt's model to stratified systems with a log normal permeability distribution. Hearn (1971) developed expressions for the pseudo relative permeability functions for communicating stratified reservoirs. Reznik et al.(1984) extended the Dykstra-Parsons method to continuous real-time basis. El-Khatib (1999) investigated the effect of crossflow on the performance of stratified reservoirs and presented a closed form analytical solution for communicating stratified systems with log-normal permeability distributions. It is interesting to know that most of these methods above assume that all layers have identical properties except permeability. Also, the time is not related explicitly to the performance. Furthermore, none of these methods considers the variation in injection rate and total pressure drop as displacement process progresses in a stratified reservoir. In this research work analytical model is developed for predicting waterflood performance in a stratified reservoir without crossflow between its adjacent layers using the main assumption of Two-phase flow (saturation gradient) existing behind the flood front. This model will account for variation in porosity, saturation, relative permeability, thickness, height in addition to permeability of different layers of the stratified reservoir.

2.0 MATERIALS AND METHODS

The linear flow model consists of series of equal thickness layers arranged in order of decreasing permeability this is illustrated by fig. 3.1 which depicts the reservoir at the time of water breakthrough in the most permeable bed.

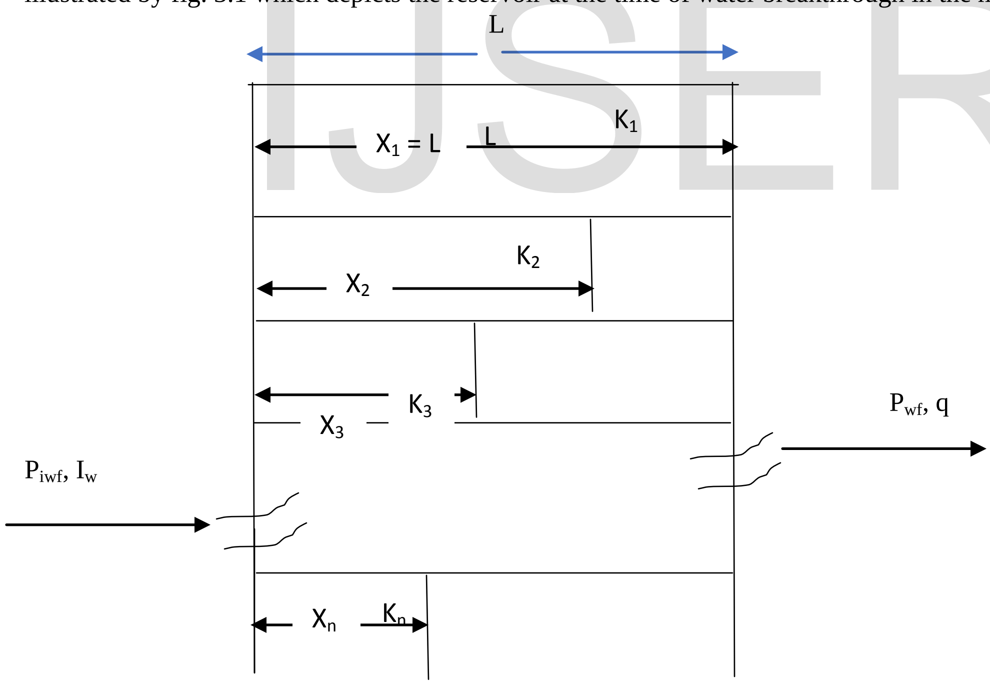

Figure 3.1: Linear Flow Model for Layer Reservoir

In order to describe water-oil flow behaviour in this stratified system above, consider it first at the time when water has advanced a distance X_1 in the most permeable layer, this is illustrated by fig 3.2

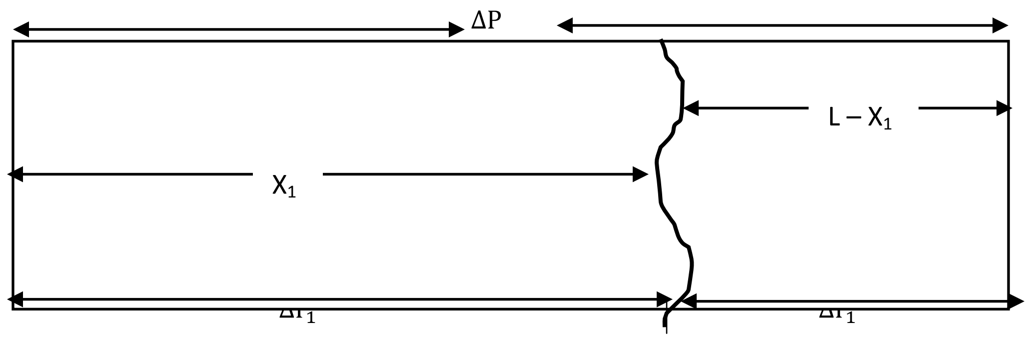

Figure 3.2: Schematic displacement front in most permeable layer

2.1 Mathematical Model Development

Assumptions of The New Model:
In order to bridge the gaps created by the existing models, the following assumptions were made for both the Non-Communicating and Communicating systems in this new analytical model:

(1) The system is linear and horizontal, the flow is incompressible, isothermal and obeys Darcy's law.
(2) Fractional flow exists behind the water displacement front instead of piston-like displacement i.e. two-phase flow exists behind the flood front before breakthrough.
(3) Highly stratified reservoir in which each layer is characterised by different Width, Length, Thickness, Permeability, Porosity and fluid Saturations.
(4) All layers have different relative permeability characteristics
(5) Constant Total water injection rate but varying injection rate into each layer of the reservoir depending on the properties of each layer.
(6) Saturation values are different for all layers i.e. S_{or} and S_{wc} and S_{oi} varies for all layer.
(7) There is initial gas saturation present at the start of the flood and no trapped gas behind the flood front.

2.1.1 Average Fluid Mobility Behind the Front

To calculate the average fluid mobility behind the front where oil and water are flowing simultaneously, consider the pressure drop behind the front across the layer ΔP_1

$$\Delta P_1 = \Delta P_{oil\ phase} + \Delta P_{water\ phase} \quad (1)$$

In terms of Darcy linear flow equation for steady state incompressible flow

$$\Delta P = \frac{i_w L \mu}{K A_1} = \frac{i_w L}{\left|\frac{\bar{K}}{\mu}\right|_1 A_1} \quad (2)$$

$$\Delta P_1 = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \quad (3)$$

$$\frac{i_w X_1}{\left|\frac{\bar{K}}{\mu}\right|_1 A_1} = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \quad (4)$$

Solving for average fluid mobility, $\left|\frac{K}{\mu}\right|$ behind the front

$$\frac{i_w X_1}{A_1}\frac{1}{\left|\frac{\bar{K}}{\mu}\right|_1} = \frac{i_w X_1}{A_1}\left[\frac{\mu_w}{K_w}\right] + \frac{i_w X_1}{A_1}\left[\frac{\mu_o}{K_o}\right] \qquad (5)$$

Therefore

$$\left|\frac{\bar{K}}{\mu}\right| = \left[\frac{\mu_w}{K_w} + \frac{\mu_o}{K_o}\right]^{-1} \qquad (6)$$

But $K_w = K_{rw}K \; and \; K_o = K_{ro}K$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1} = \bar{\lambda}_t \qquad (7)$$

Where $\bar{\lambda}_t$ is the average total two phase mobility behind the front. Equation (7) gives the average total fluid mobility behind the front at distance X_1 as shown in fig.3.2

2.1.2 Average Fluid Mobility Ahead of the Displacement Front.

Consider the pressure drop across the distance ($L – X_1$) ahead of the front in fig 3.2

$$\Delta P_2 = \Delta P_{oil\ phase} + \Delta P_{water\ phase} \qquad (8)$$

Using Darcy's equation

$$\Delta P_2 = \frac{i_w(L - X_1)}{\left|\frac{\bar{K}}{\mu}\right| A_1} \qquad (9)$$

$$\Delta P_{oil\ phase} = \frac{i_o\mu_o(L - X_1)}{|K_o| A_1} \qquad (10)$$

$$\Delta P_{water\ phase} = \frac{i_w\mu_w(L - X_1)}{|K_w| A_1} \qquad (11)$$

$$\frac{i_w(L - X_1)}{\left|\frac{\bar{K}}{\mu}\right| A_1} = \frac{i_w\mu_w(L - X_1)}{K_w A_1} + \frac{i_o\mu_o}{K_o A_1}(L - X_1) \qquad (12)$$

$$\left|\frac{\bar{K}}{\mu}\right| = \left[\frac{\mu_w}{K_w} + \frac{\mu_o}{K_o}\right]^{-1} \qquad (13)$$

But $K_w = K_{rw}K \; and \; K_o = K_{ro}K$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1} \qquad (14)$$

Since it is assumed that no water is flowing ahead of the front, therefore $K_{rw} = 0$

$$\left|\frac{\bar{K}}{\mu}\right| = K_1 \left[\frac{\mu_o}{K_{ro}}\right]^{-1} \qquad (15)$$

Equation (15) gives the average total fluid mobility ahead of the front of distance ($L - X_1$).

2.1.3 Two Phase Mobility Ratio

Mobility ratio can be defined mathematically as

$$m = \frac{\lambda_{displacing\ fluid\ flowing\ behind\ the\ front}}{\lambda_{displacing\ fluid\ flowing\ ahead\ the\ front}} \qquad (16)$$

But it was assumed that oil and water flow simultaneously behind the front and oil only ahead of the front, then we have Two-phase mobility ratio m_{tp}

$$m_{tp} = \frac{\bar{\lambda}_t}{\lambda_o} = \frac{\overline{\lambda_o + \lambda_w}}{\lambda_o} \qquad (17)$$

Where

$\bar{\lambda}_t$ = average total two phase mobility behind the front evaluated at average water saturation behind the front

$$m_{tp} = \frac{\left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1}}{\left[\frac{\mu_o}{K_{ro}}\right]^{-1}} \qquad (18)$$

Therefore,

$$m_{tp} = \left[\frac{\mu_w}{K_{rw}} + \frac{\mu_o}{K_{ro}}\right]^{-1} \left[\frac{\mu_o}{K_{ro}}\right] \qquad (20)$$

Equation (19) and (20) gives the two-phase mobility ratio.

2.1.4. Position of Flood Front

Consider figure 3.2; The total pressure drop across this layer is:

$$\Delta P = P_{iwf} - P_{wf} = \Delta P_1 + \Delta P_2 \qquad (21)$$

In terms of Darcy's linear flow

$$\Delta P_1 = \frac{i_w \mu_w X_1}{K_w A_1} + \frac{i_o \mu_o X_1}{K_o A_1} \qquad (22)$$

$$\Delta P_2 = \frac{i_w \mu_o (L - X_1)}{K_o A_1} \qquad (23)$$

$$\Delta P = \frac{i_w L}{\left|\frac{\bar{K}}{\mu}\right| A_1} \qquad (24)$$

Substituting Equation 22 -24 into equation into equation 21 and solving for average mobility in the layer

$$\left|\frac{\bar{K}}{\mu}\right| = K_1 L \left[\frac{\mu_w X_1}{K_{rw}} + \frac{\mu_o X_1}{K_{ro}} + \frac{\mu_o (L - X_1)}{K_{ro}}\right]^{-1} \qquad (25)$$

Therefore, the average injection flux is:

$$U_1 = \frac{i_w}{A_1} = \frac{\left|\frac{\bar{K}}{\mu}\right|_1 \Delta P}{L} \qquad (26)$$

$$U_1 = K_1 \Delta P \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \qquad (27)$$

The actual velocity of the flood front is given by the expression

$$V_1 = \frac{dx_1}{dt} = \frac{U_1}{\phi_1 \Delta S_{w1}} \qquad (28)$$

Where ΔS_{w1} represent the change in water saturation across the front. Therefore

$$\frac{dx_1}{dt} = \frac{K_1 \Delta P}{\phi_1 \Delta S_{w1}} \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \qquad (29)$$

And $\Delta P = constant = \frac{\phi_1 \Delta S_{w1} \frac{dx_1}{dt}}{K_1} \left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o}{K_{ro1}} X_1 + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]$

Similarly, for second layer,

$$V_2 = \frac{K_2 \Delta P}{\phi_2 \Delta S_{w2}} \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}}(L - X_2)\right]^{-1}$$

$$\Delta P = constant = \frac{\phi_2 \Delta S_{w2} \frac{dx_2}{dt}}{K_2} \left[\frac{\mu_w X_2}{K_{rw2}} + \frac{\mu_o}{K_{ro2}} X_2 + \frac{\mu_O}{K_{ro2}}(L - X_2)\right] \qquad (30)$$

Equating equation 28 and 30 and

$$let\ B_1 = \frac{\mu_w}{K_{rw1}} + \frac{\mu_o}{K_{ro1}}, B_2 = \frac{\mu_w}{K_{rw2}} + \frac{\mu_o}{K_{ro2}}, D_1 = \frac{\mu_o}{K_{ro1}}, D_2 = \frac{\mu_o}{K_{ro2}}$$

$$\phi_1 \Delta S_{w1} dx_1 K_2 [B_1 X_1 + D_1 (L - X_1)] = \phi_2 \Delta S_{w2} dx_2 K_1 [B_2 X_2 + D_2 (L - X_2)] \qquad (33)$$

Rearranging the above expression

$$K_2 \phi_1 \Delta S_{w1} [B_1 X_1 + D_1 (L - X_1)] dx_1 = K_1 \phi_2 \Delta S_{w2} [B_2 X_2 + D_2 (L - X_2)] dx_2 \qquad (34)$$

Integrating

$$K_2 \phi_1 \Delta S_{w1} \int_0^L [B_1 X_1 + D_1 (L - X_1)] dx_1 =$$

$$K_1 \phi_2 \Delta S_{w2} \int_0^{x_2} [B_2 X_2 + D_2 (L - X_2)] dx_2 \qquad (35)$$

$$let\ A = \frac{K_2 \phi_1 \Delta S_{w1}}{K_1 \phi_2 \Delta S_{w2}} and\ \Delta S_{wi} = \bar{S}_{wi} - S_{wci} \qquad (41)$$

$$A[B_1 L^2 - D_1 L^2] = [X_2^2 (B_2 - D_2) + 2 D_2 L X_2] \qquad (43)$$

Rearranging equation (3.43) and divide through by L^2

$$(B_2 - D_2)\left[\frac{X_2^2}{L^2}\right] + 2D_2\left[\frac{X_2}{L}\right] - A(B_1 + D_1) = 0 \qquad (45)$$

Solve equation (45) above to obtain:

$$\frac{x_2}{L} = \frac{D_2 \pm \left({D_2}^2 + (B_2 - D_2)(B_1 + D_1)A\right)^{1/2}}{(D_2 - B_2)} = X_2 \qquad (46)$$

Therefore, in general the fractional distance the flood front has moved through layer 2 at the front has moved in bed n is:

$$\frac{X_n}{L} = \frac{D_n \pm \left({D_n}^2 + (B_n - D_n)(B_1 + D_1)A\right)^{1/2}}{(D_n - B_n)} \qquad (47)$$

2.1.5. Water Injection Rate

Using figure 3.2, consider total pressure drop through this layer, ∆P, the fluid injection rate into this layer can be obtained using the Darcy's flow equation

$$\Delta P = \frac{i_w \mu L}{KA} \qquad (48)$$

$$i = \frac{KA_1\Delta P}{L} \qquad (49)$$

Considering the average mobility in the layer

$$i_{total} = \frac{\left|\frac{\bar{K}}{\mu}\right|_1 A_1\Delta P}{L} \qquad (50)$$

$$Since \left|\frac{\bar{K}}{\mu}\right| = K_1 L\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \qquad (51)$$

Substitute equation (51) into equation (50)

$$i_{total\,1} = K_1 A_1 \Delta P\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]^{-1} \qquad (52)$$

$$= \frac{K_1 A_1 \Delta P}{\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]} \qquad (53)$$

$$multiply\ through\ by\ \frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o} \qquad (54)$$

$$i_{total\,1} = \frac{K_1 A_1 \Delta P\left[\frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o}\right]}{\left[\frac{\mu_w X_1}{K_{rw1}} + \frac{\mu_o X_1}{K_{ro1}} + \frac{\mu_O}{K_{ro1}}(L - X_1)\right]} \qquad (55)$$

Substitute equation (20) into equation (55)

$$i_{total\,1} = \frac{K_1 A_1 \Delta P\left[\frac{K_{rw1}}{\mu_w} + \frac{K_{ro1}}{\mu_o}\right]}{X_1 + (L - X_1)m_{tp}} \qquad (56)$$

Divide through by $\frac{m_{tp}}{m_{tp}}$

$$i_{total\,1} = \frac{K_1 A_1 \Delta P \frac{K_{ro1}}{\mu_o}}{\frac{X_1}{m_{tp}} + (L - X_1)} \quad (57)$$

$$= \frac{K_1 A_1 \Delta P \lambda_o}{\frac{X_1 + m_{tp} L - m_{tp} X_1}{m_{tp}}} \quad (58)$$

$$i_{total\,1} = \frac{K_1 A_1 \Delta P \lambda_o}{\frac{m_{tp} L - X_1 (m_{tp} - 1)}{m_{tp}}} \quad (59)$$

$$i_{total\,1} = \frac{K_1 A_1 \Delta P \lambda_o}{L\left(1 - \left(\frac{m_{tp} - 1}{m_{tp}}\right) X_1\right)} \quad (60)$$

Therefore:

$$i_{wi} = \frac{k_i A_i \lambda_0^0 \Delta P_t}{L(1 - (\frac{m_{tp} - 1}{m_{tp}}) X_i} \quad (61a)$$

2.1.6. Two-Phase Vertical Sweep Efficiency E_{VT}

The vertical sweep efficiency, $\mathbf{E_{VT}}$, is defined as the fraction of the vertical section of the pay zone that is contacted by the injection fluid, which depends primarily on (1) the mobility ratio and (2) total volume injected. As a consequence, there is need to developed new correlation for Two-Phase Vertical Sweep Efficiency for a stratified reservoir since the two existing traditional methods assumes Piston-like displacement and use the end point mobility ratio in their model formulation. This new model employs the concept of recoverable oil from a layer to derive expression for vertical sweep efficiency.

Recoverable oil from layer 1 is:

$$PV_1 (1 - S_{or1} - S_{wc1}) \quad (61b)$$

But the fraction of recoverable oil from layer one at breakthrough is:

$$\frac{PV_1 (\bar{S}_{w1} - S_{wc1})}{PV_1 (1 - S_{or1} - S_{wc1})} \quad (62)$$

$$= \frac{(\bar{S}_{w1} - S_{wc1})}{(1 - S_{or1} - S_{wc1})} \quad (63)$$

But at the time of breakthrough in layer 1, layer 2 is not at breakthrough and is at a fractional distance of X_2 and X_3 for layer 3 and so on, therefore;

The vertical coverage by the water front, E_V, in any layer the time of breakthrough in the most permeable bed is given as:

$$Ev_n = \frac{X_n PV_n (\overline{S\mathrm{wn}} - S_{wc})}{PV_n (1 - S_{or} - S_{wc})} \quad (64)$$

Recall;

$$\frac{x_n}{x_i} = \frac{D_n \pm \left(D_n{}^2 + (B_n - D_n)(B_i + D_i)A\right)^{1/2}}{(D_n - B_n)} = X_n \qquad (65)$$

In any breakthrough layer, the position of flood front $X_n = 1$

Thus, if there are N-layers, vertical sweep efficiency for layer 1 at breakthrough will be calculated:

$$E_{v1} = \frac{PV_1 X_1(\bar{S}_{w1} - S_{wc1})}{PV_1(1 - S_{or1} - S_{wc1})} = \frac{(\bar{S}_{w1} - S_{wc1})}{(1 - S_{or1} - S_{wc1})} \qquad (66)$$

Also for n layer,

$$E_{vn} = \frac{PV_n X_n(\bar{S}_{wn} - S_{wcn})}{PV_n(1 - S_{orn} - S_{wcn})} \qquad (67)$$

Where X_n is the fractional distance travelled by displacement front in layer n.

But for multilayer reservoir:

The vertical coverage (vertical sweep efficiency) by the water front, E_V, at the time of breakthrough in bed 1 is given as:

$$E_{V1} = \frac{\sum_{j=1}^{i} \frac{(\bar{S}_{w1} - S_{wc1})}{(1 - S_{or1} - S_{wc1})} + \sum_{j=i+1}^{n} \frac{X_j\left(\bar{S}_{wj} - S_{wcj}\right)}{\left(1 - S_{orj} - S_{wcj}\right)}}{n} \qquad (68)$$

2.1.7 Two-Phase Displacement Efficiency E_{DT}

The displacement efficiency ED is the fraction of movable oil that has been displaced from the swept zone at any given time or pore volume injected.
Or

$$E_{DT} = \frac{volume\ of\ oil\ at\ start - remaining\ oil\ volume}{volume\ of\ oil\ at\ the\ start\ of\ the\ flooding} \qquad (70)$$

Or

$$E_{DT} = \frac{\frac{S_{oi}}{B_{oi}} - \frac{\bar{S}_o}{B_{oi}}}{\frac{S_{oi}}{B_{oi}}} \qquad (71)$$

Where S_{oi}=initial oil saturation at the start of the flood

B_{oi}= oil FVF at the start of flood, bbl/STB

$\bar{S}_o$= average oil saturation in the flood pattern at a particular point during the flood.

Assuming a constant oil formation volume factor, equation above reduced to

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$$E_{DT} = \frac{S_{oi} - \bar{S}_o}{S_{oi}} \qquad (72)$$

Where the initial oil saturation $S_{oi}$is given by:

$$S_{oi} = 1 - S_{wc} - S_{gi}$$

However, in the sept area, the gas saturation is considered zero, thus:

$$\bar{S}_o = 1 - \bar{S}_w$$

The displacement efficiency E_{DT} can be expressed more conveniently in terms of water saturation by substituting the above relationships into Equation (5) to give:

$$E_{DT} = \frac{\bar{S}_w - S_{wc} - S_{gi}}{1 - S_{wc} - S_{gi}} \qquad (73)$$

Where:

$\bar{S}_w$= average water saturation in the swept area.
S_{gi}= initial gas saturation at the start of the flood.
S_{wc}= initial water saturation at the start of the flood.

2.2 Modelling of Displacement Performance

Since the main objective of displacement performance is to estimate the volume of oil production, and the volume of water that must be handled per volume of oil, once production begins. The expressions that can be used to calculate those quantities are developed below for both performances to breakthrough and after breakthrough, since the layers are assumed to be completely separated by impermeable thin strata so that no crossflow takes place between different layers

2.2.1 Performance from Fill-up to Breakthrough

The end of gas filled up period mark the beginning of secondary oil production. It is assumed that, on a reservoir volume basis, the total oil producing rate during this stage is equal to water injection rate.

Thus, the oil producing rate in STB/D is:

$$q_o = \frac{I_w}{B_o}$$

Estimate the cumulative water injected at fill-up for the reservoir, W_{If}(rb)

$$W_{If} = (PV_T)S_{gi} \qquad (75a)$$

Since total injection rate is constant $I_{wf} = I_{wT}$

Therefore, time to fill-up

$$t_f = \frac{W_{If}}{I_{wf}} \qquad (75b)$$

The cumulative oil production, N_p, since the beginning of fill-up can be computed in terms of cumulative water injected during fill-up as:

$$N_p = \frac{W_I - W_{If}}{B_o} \qquad (75c)$$

Note that it has been assumed that after fill up to breakthrough, the injection rate is constant as I_{wbt}

2.2.2 Performance at Breakthrough

a. Oil Displaced

Until the water arrives at the end of a system, oil will be produced at the same rate as water injected for incompressible system where interstitial water is assumed to be immobile. When water breakthrough occurs, a water saturation gradients exist from inlet to the end of the system. The volume of water in the system between x = x_1 and x = x_2 can be obtained by integration

$$V_w = \int_{x_1}^{x_2} S_w A \emptyset \, \partial x \qquad (76a)$$

Where V_w is the volume of water in the porous rock between x_1 and x_2. The volume of oil displaced from the region is:

$$V_o = V_w - A\emptyset(x_2 - x_1)S_{wi} \qquad (76b)$$

Where V_o is the volume of oil displaced from the interval $x_1 \leq x \leq x_2$

To get the average water saturation for this region ($x_1 \leq x \leq x_2$), we need to solve equation $(76a)$ above.

Let $\overline{S_w}$ represent the volumetric average water saturationfor the region $x_1 \leq x \leq x_2$, then

$$\bar{S}_w = \frac{\int_{x_1}^{x_2} S_w A \emptyset \, \partial x}{\int_{x_1}^{x_2} A \emptyset \, \partial x} \qquad (77)$$

For constant values of $\emptyset$ and A, equation(77)reduces to

$$\bar{S}_w = \frac{\int_{x_1}^{x_2} S_w \partial x}{x_2 - x_1} \qquad (78)$$

The integral in equation (78) can be evaluated by the use of

$$x_{sw} = \frac{q_t t}{A\emptyset}\left(\frac{df_w}{dS_w}\right)_{sw} \qquad (79)$$

The derivative of the product xS_w is expressed as

$$d(xs_w) = S_w dx + xdS_w \qquad (80)$$

The integrand $S_w$dx = d (xS_w – xdS_w)

Substitution into equation (75)with corresponding changes of integration limit yields

$$\bar{s}_w = \frac{1}{x_2 - x_1}\int_1^2 \mathrm{d}(\mathrm{x}S_w - \mathrm{xd}S_w) \qquad (81)$$

$$\bar{s}_w = \frac{1}{x_2 - x_1}\int_{x_1 s_{w1}}^{x_2 s_{w2}} \mathrm{d}(\mathrm{x}S_w) - \frac{1}{x_2 - x_1}\int_1^2 \mathrm{xd}S_w \qquad (82)$$

and

$$\bar{s}_w = \frac{x_2 s_2 - x_1 s_{w1}}{x_2 - x_1} - \frac{1}{x_2 - x_1}\int_1^2 \mathrm{xd}S_w \qquad (83)$$

Now consider the remaining in equation (83) from equation (82) we have

$$\int_1^2 \mathrm{xd}S_w = \frac{q_t t}{A\emptyset}\int_1^2 df_w \qquad (86)$$

Therefore

$$\int_1^2 \mathrm{xd}S_w = \frac{q_t t}{A\emptyset}(f_{w2} - f_{w1}) \qquad (87)$$

Thus, the expression for the average water saturation for the interval $x_1 \leq x \leq x_2$ is given by

$$\bar{s}_w = \frac{x_2 s_2 - x_1 s_{w1}}{x_2 - x_1} - \left(\frac{q_t t}{A\emptyset}\right)\frac{(f_{w2} - f_{w1})}{x_2 - x_1} \qquad (88)$$

When x_1 = 0 and sufficient time has passed for water to arrive at the end of the core (x_2 = L), the average water saturation in the core is:

$$\bar{s}_w = s_{w2} - \left(\frac{q_t t}{A\emptyset L}\right)(f_{w2} - f_{w1}) \qquad (89)$$

Usually, f_{w1}= 1.0 at x = 0 and equation (3.89) becomes:

$$\bar{s}_w = s_{w2} + \left(\frac{q_t t}{A\emptyset L}\right)(1 - f_{w2}) \qquad (90)$$

Note that $q_t t$ represents the total volume of water injected (w_i) while the $A\emptyset L$ is the pore volume of porous rock PV. Therefore, we define pore volume of water injected as

$$Q_i = \frac{w_i}{A\emptyset L} \qquad (91)$$

For constant injection rate,

$$Q_i = \frac{q_t t}{A\emptyset L} \qquad (92)$$

Substituting into equation (3.90)

$$\bar{s}_w = s_{w2} + Q_i(1 - f_{w2}) \qquad (93)$$

Alternatively

At the end of the system (x = L), the water saturation is S_{w2} after water arrives, becomes

$$x_{sw2} = L = \frac{q_t t}{A\emptyset}\left(\frac{df_w}{dS_w}\right)_{Sw2} \qquad (94)$$

Or

$$Q_i = \frac{1}{\left(\frac{df_w}{dS_w}\right)_{Sw2}} \qquad (95)$$

So, that equation (3.93) becomes

$$\bar{s}_w = s_{w2} + \frac{(1 - f_{w2})}{f'_{s_{w2}}} \qquad (96)$$

Since the initial hydrocarbon in place at the start of the flood is

$$N_p = PV(S_{oi}) \qquad (97)$$

But $S_{oi} = 1 - S_{wc}$

Therefore

$$N_p = PV(1 - S_{wc}) \qquad (98)$$

since oil displaced from the reservoir is a function of the Areal Sweep Efficiency and the Vertical Sweep Efficiency. Therefore

$$N_p = PV(1 - S_{wc}) * E_{DT} * E_{VT} * E_A \qquad (99)$$

But for this modeling, the correlation of Willhite (1986) will be used for calculating E_A

$$E_A = 0.54602036 + \frac{0.03170817}{M} + \frac{0.30222997}{e^M} - 0.00509693M \qquad (100)$$

Also, E_{VT} and E_{DT} is given by equation (68) and (74).Substituting equation (68) and (74) into equation (99)

$$N_p = PV(1 - S_{wc}) * \frac{\bar{S}_w - S_{wc}}{1 - S_{or} - S_{wc}} * \frac{\bar{S}_w - S_{wc}}{1 - S_{wc}} * E_A \qquad (101)$$

Therefore

$$N_p = \frac{PV(\bar{S}_w - S_{wc})^2 E_A}{1 - S_{or} - S_{wc}} \qquad (102)$$

In general, for i layer at breakthrough

$$N_{pi} = \frac{PV_i(\bar{S}_{wi} - S_{wci})^2 E_{Ai}}{1 - S_{ori} - S_{wci}} \qquad (103)$$

or

$$N_{pi} = \frac{PV_i(\bar{S}_{wi} - S_{wci})^2 E_{Ai}}{MOV_i} \qquad (104)$$

Where MOV = movable oil from layer i`

Note that if the initial gas saturation is present:

$$N_p = PV\left(1 - S_{wc} - S_{gi}\right) * \frac{\bar{S}_w - S_{wc} - S_{gi}}{1 - S_{or} - S_{wc} - S_{gi}} * \frac{\bar{S}_w - S_{wc} - S_{gi}}{1 - S_{wc} - S_{gi}} * E_A$$

Therefore:

$$N_p = \frac{PV\left(\bar{S}_w - S_{wc} - S_{gi}\right)^2 E_A}{(1 - S_{or} - S_{wc} - S_{gi})} \qquad (105)$$

3. RESULTS AND DISCUSSION

The developed models are applied to predict the waterflood performance of ten layers reservoir, and the results of the model was compared with the 17 years waterflood production and injection history of the field in other to validate the new model. The model results were also compared with one of the most commonly used existing waterflood prediction old model. The data showing the layer's characteristic for the case studied reservoir is given in Table 1. The results obtained are as shown in figs.3 through 14.

Table 1: Characteristics Of 10 Layers Stratified Reservoir

Characteristics/Layer	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Permeability, k, [md]	1000	795.0	500	432.0	348.5	280.5	230.0	188.0	149.0	110.0
Oil End point relative permeability, kroe	0.85	0.9	0.75	0.8	0.75	0.75	0.8	0.68	0.80	0.85
Water End point relative permeability, krwe	0.35	0.5	0.7	0.4	0.35	0.37	0.23	0.2	0.28	0.3
Porosity, (ϕ_1), %	25	25	25	25	25	25	25	25	25	25
Initial oil saturation, Soi	80	70	70	75	80	75	70	85	60	80

Connate water saturation, Swc	20	30	30	25	20	25	30	15	40	20

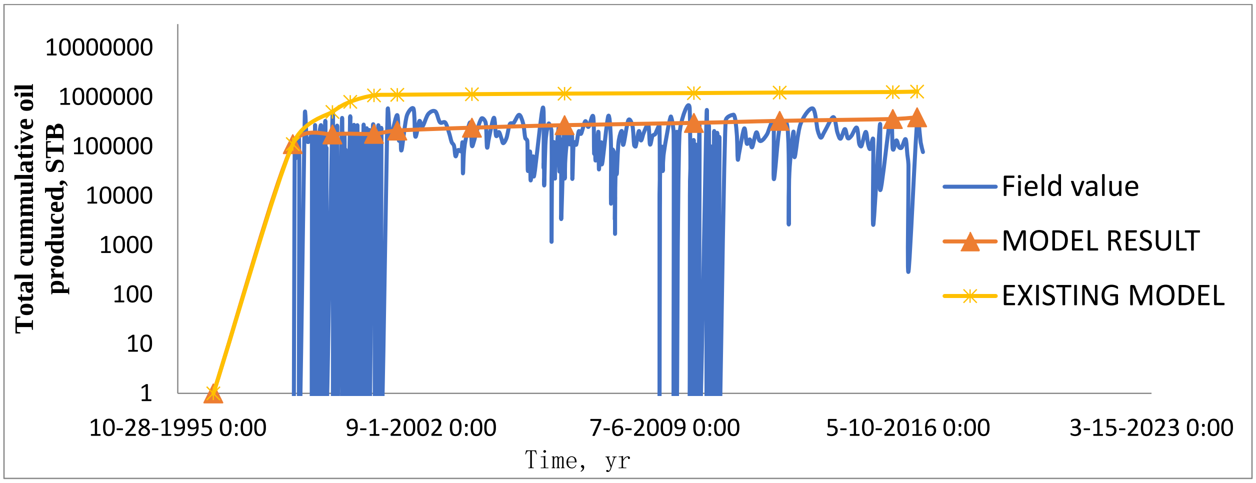

Figure 3. Cumulative Oil Produce Performance Curve

Cumulative Oil Recovery

The Cumulative oil recovery performance comparison of the new model, field and old existing model results is as shown in fig. 3. It can be seen that the cumulative oil production performance curve of the new model is in good agreement with the real field cumulative oil production performance curve. The new model prediction output closely fit with that of the field and can actually serve as oil recovery predicting tool for the case study reservoir. The accuracy is due to the new method used to estimate the areal and vertical sweep efficiency in the new model oil recovery equation. However, a significant difference was observed when the field performance was compared with old existing model, this further confirms the accuracy the new analytic model.

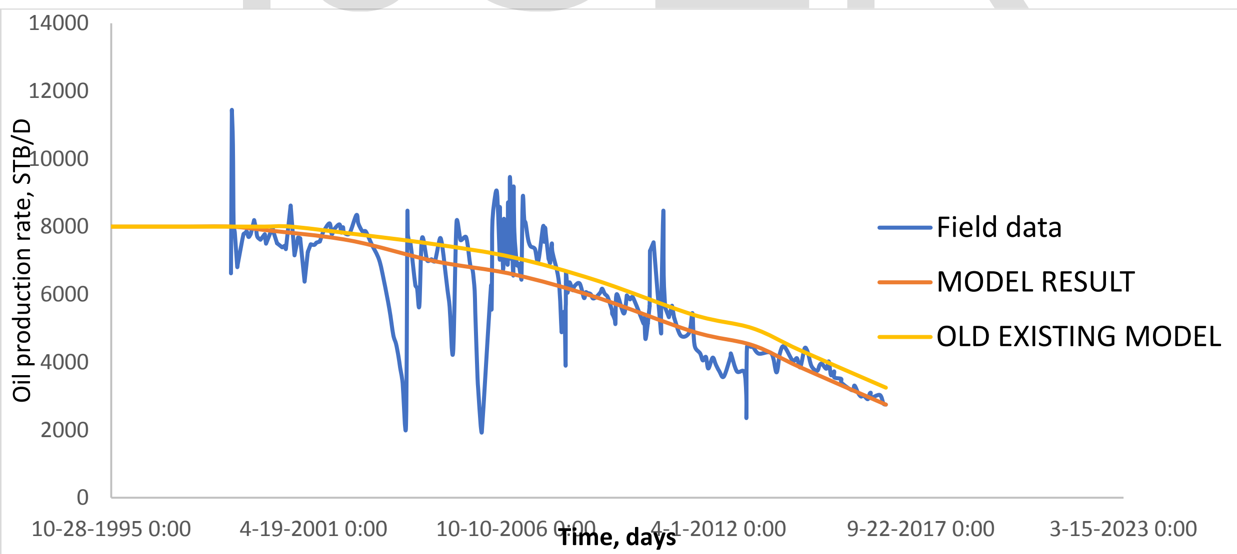

Figure 4. Total Oil Production Rate Performance Curve

Oil Production Rate

Figure 4 presents comparison of total oil production rate performance.it can be seen that the rate of producing oil predicted by the new model closely fit with that of the field history, with an insignificant difference, as the water injection continues, the total oil production rate continues to decrease due to higher volume of water injected, this trend is also followed by the model performance curve as shown in fig. 4. An over-estimation of the oil rate was seen in in the performance of the most commonly used model.

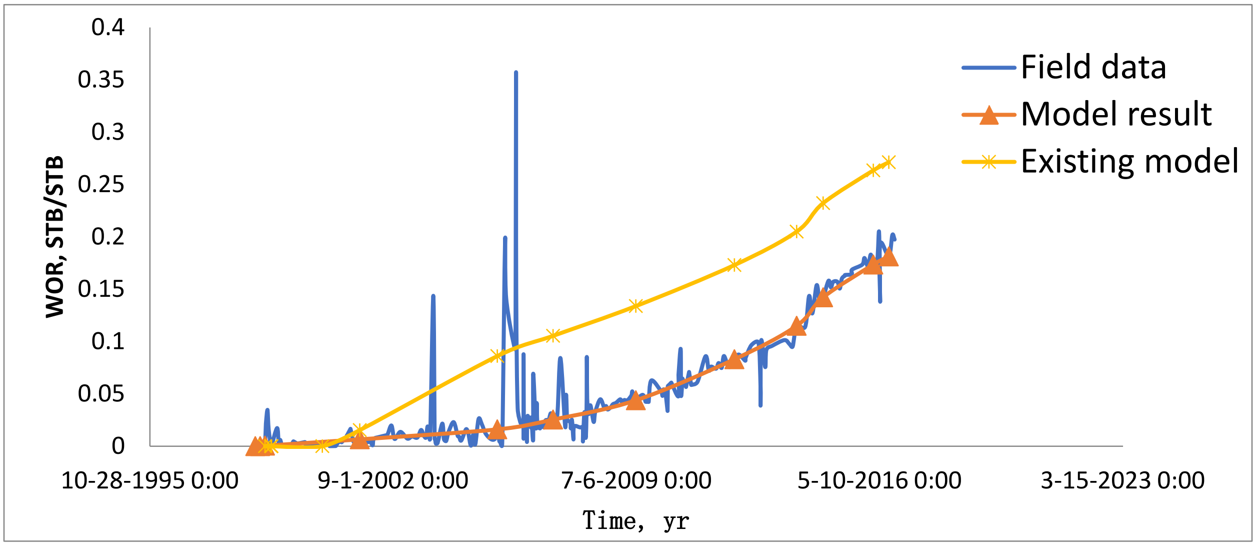

Figure 5. Total Water Oil Ratio Performance Curve

Water Oil Ratio

Figure 5 presents the water oil ratio performance curve of the reservoir under study, the predicted performance result of the newly developed model was compared with field waterflood operation and most commonly used existing model performance. It was observed that the predicted water oil ratio of the new model closely agrees with the filed observed data**,** while there is a wide difference value when compared with most common existing model performance as shown in figure 5.

Water Production Rate

Figure 6 show the Total Water Production Rate Performance for field, model result and an existing model. It can be seen that the water flow rate of the new model actually tallies with the field history oil flow rate, which confirms the accuracy of the new model. However, there is a significant difference between the field water rate and that of commonly used existing model, this therefore makes the new model preferable.

Water produce

Figure 7 presents the cumulative water produced for the new model in comparison with field performance and old existing model, it can be seen that the new model approximately fit with the field data while there is a wide gap between the field performance and old existing model. With this result, it can be concluded that the new model result can accurately predict future water produce for the reservoir under study.

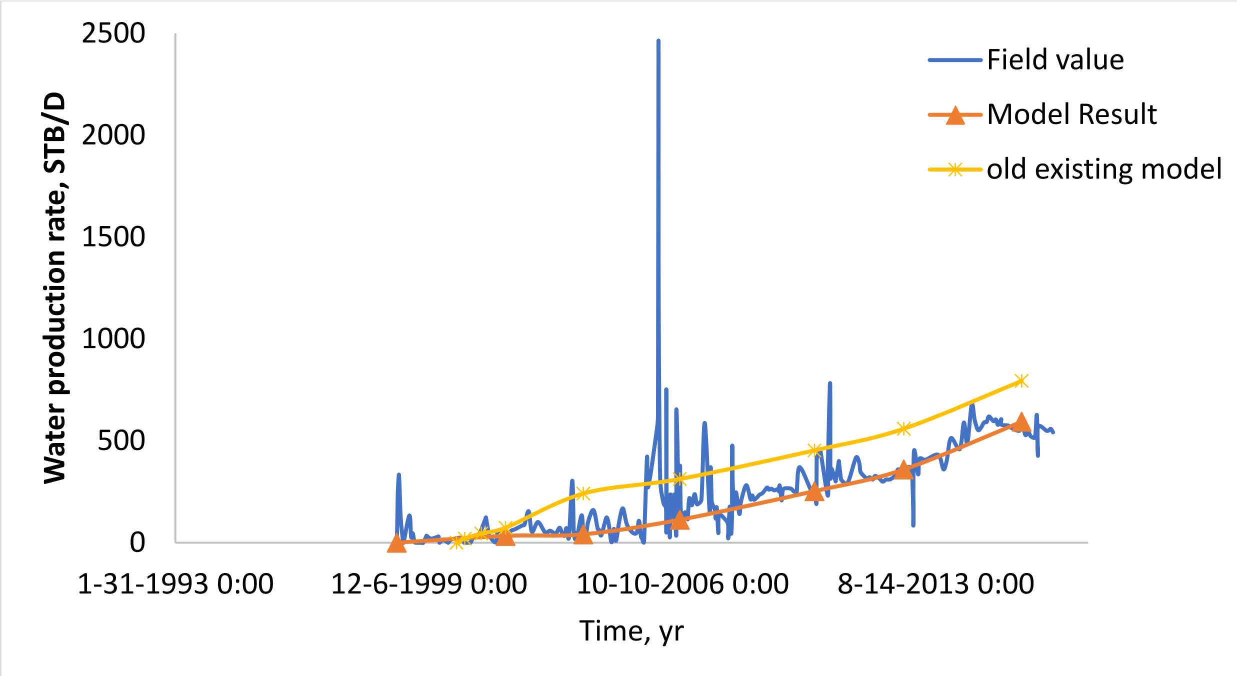

Figure 6. Total Water Production Rate Performance Curve

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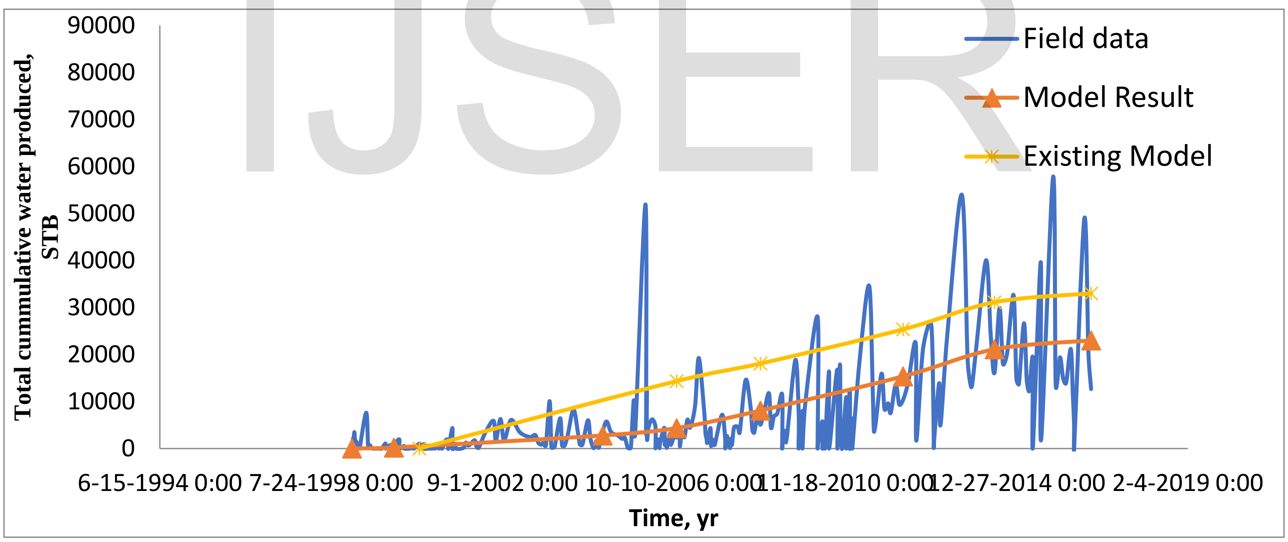

Figure 7. Total Cumulative Water Produced Performance Curve

Injection rate

Fig 8 shows the injection rate performance of the new model when applied to the cased studied reservoir. It can be seen that the model injection rate actually follows the same trend as the field injection performance, this is due to the fact that the new model takes into account the changes in injectivity ratio as displacement progresses as compare to other existing model

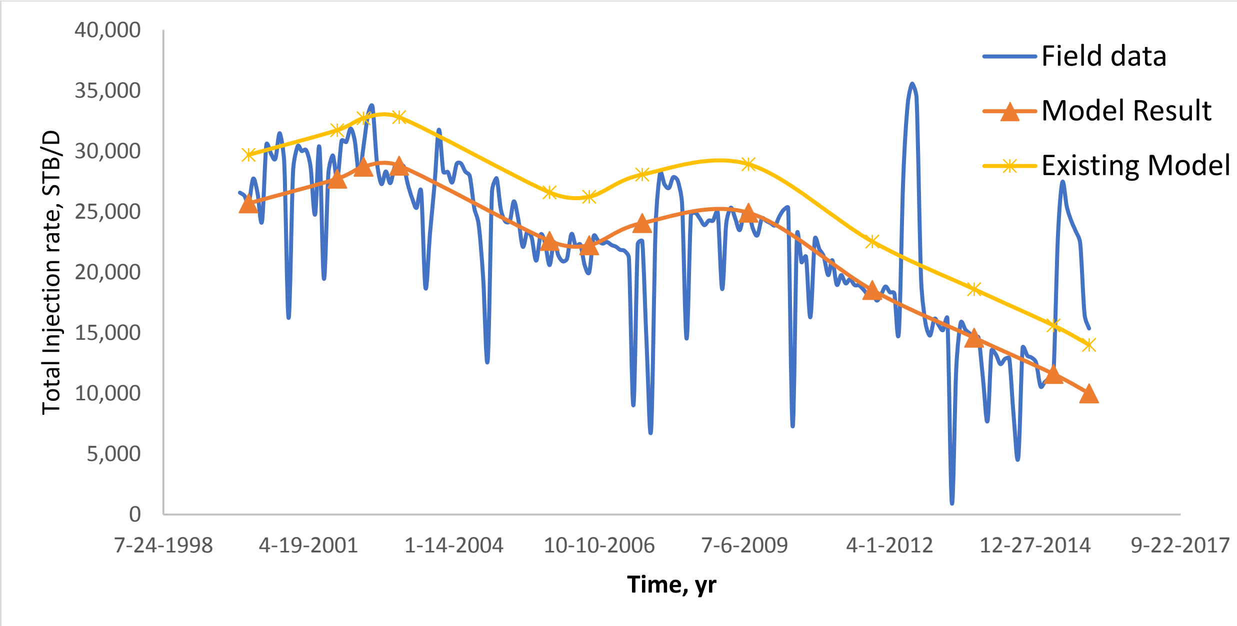

Figure 8. Total Injection Rate Performance Curve.

4.0 CONCLUSION

1. A new mobility ratio model that account for the presence of resistance to flow contributed by two fluids of different saturation (saturation gradient) behind the flood front and average total fluid mobility in the invaded zone has been developed
2. A mathematical model is developed for predicting reservoir layer injection rate for a stratified reservoir taking into account two-phase (fractional) flow behind the flood front of each layer, physical properties of each layer of the stratified system, fluid mobilities ahead and behind the flood front for each layer, Bottom-hole injection pressure, producing well pressure, and average reservoir pressure at the start of injection and flood front advancement in successive layers of the reservoir at a real point in time. This model gives accurate that actually fit with the field data.
3. A new method of estimating vertical and areal weep efficiency of a linear waterflood in a stratified reservoir has been developed
4. A new mathematical model was developed to predict the waterflooding performance in a linear stratified reservoir without crossflow. The model assumes no particular permeability distribution and account for variation in other rock properties such as porosity, fluid saturation, relative permeability and other physical properties of the rock.

NOMENCLATURE

B_{oi} = formation volume factor for oil layer i, RB/STB

B_{wi} =for formation volume factor for water layer i, RB/STB

$I_{wi} = I_{total\ i}$ =The injection rate into layer i at breakthrough, STB/D

$K_i = A$bsolute permeability for layer i, md

K_{roi}= relative permeability of layer i to water

K_{roi}= relative permeability of layer i to oil

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L =distance between injector and producer, ft

M_{tp} = Two − Phase Mobility Ratio at breakthrough, fraction

ΔP_t = difference between injection pressure and producing well bottom hole pressure, psi

P_{wf} =bottom hole pressure of the producer well, Psi

P_{inj} =injection pressure at the injector well, Psi

S_{wci} = connate or initial water saturation for layer i, fraction,

ΔS_{wi} = change in water saturation across the displacement front for layer i

S_{ori} = residual oil saturation in layer i, fraction

S_{wci} =initial or connate water saturation in layer i, fraction

S_{oi} = initial oil saturation in layer i, fraction

S_{wi} = water saturation in layer i at a particular time

$\bar{s}$ = Average saturation in the swept area, fraction

x_i = distance travel by displacement front in layer i, ft

$X_i = \left(\frac{x_1}{L}\right)$ =, fractional distance travel by displacement front in layer i , fraction

λ_0^0 =, mobility of oil at end point water saturation(Swc), cp^{-1}

$\mu_w = viscosity\ of\ water$, cp

$\mu_o = viscosity\ of\ oil$, cp

ϕ =porosity, fraction

$\bar{\lambda}_t$ = is the average total two phase mobility behind the front.

Subscript

i = layer under consideration

$n = i + 1$

2 = as for layer 2

bt = at breakthrough

D = dimensionless

o = oil

r = relative

t = total

w = water

REFERENCES

[1] Buckley, S. E. and Leverett, M. C. (1942): "Mechanism of Fluid Displacement in Sands," *Trans., AIME* 146, 107-116.

[2] Carll, J.F. (1880): The Geology of the OH Regions of Warren, Venango, Clarion, and Butler Counties, Pennsylvania, Second Geological Survey of Pennsylvania III, pp. 1875-1879.

[3] Caudle, B. H. and Witte, M. D. (1959): "Production Potential Changes during Sweep-out in a Five-Spot Pattern,"Trans, AIME 216, 446-448.

[4] Dykstra, H. and Parsons, H. L. (1950): "The Prediction of Oil Recovery by Waterflooding," Secondary Recovery of Oil in the United States, 2nd ed., API, New York 160-174.

[5] Dykstra-Parsons (1984) "Vertical Stratification Discrete Solution to a Continuous, Real-Time Basis," *SPE Journal*, (December, 1984) 643.

[6] El-Khatib, N. (1999). Waterflooding Performance of Communicating Stratified Reservoirs with Log-Normal Permeability Distribution. *SPE Res Eval & Eng* **2** (6): 542-549. SPE-59071-PA. http://dx.doi.org/10.2118/59071-PA

[7] Fettke, C.R. (1938): "Bradford Oil Field, Pennsylvania and New York," Pennsylvania Geological Survey, 4th Series M-21.

[8] Hearn, C.L. (1971): ''Simulation of Stratified Waterflooding by Pseudo Relative Permeability Curves'', *JPT* (July 1971) 805.

[9] Muskat, M. (1950): 'The Effect of Permeability Stratification In Complete Water-Drive Systems,'' *Trans.*, AIME 189, 349.

[10] Stiles, W. E. (1949): "Use of Permeability Distribution in Waterflood Calculations," Trans., *AIME* (1949) 186, 9-13.

[11] Warren, J.E., and Cosgrove, J.J. (1964),: "Prediction of Waterflood Behavior in a Stratified System," *SPE Journal*, (June, 1964), 149.

[12] Willhite, G.P. (1986) "Waterflooding", Textbook Series, SPE, Dallas

[13] Hiatt, W. N. (1958): "Injected-Fluid Coverage of Multi-Well Reservoirs with Permeability Stratification", *Drilling and Production Practices*, San Antonio, 165.

[14] Stiles, W. E. (1949): "Use of Permeability Distribution in Waterflood Calculations," Trans., *AIME* (1949) 186, 9-13

[15] Prats, M., Matthews, C. S., Jewett, R. L. and Baker, J. D. (1959) "Prediction of Injection Rate and Production History for Multi-Fluid Five-Spot Floods," *Trans., AIME* (1959) 216, 98-105.

[16] Reznik,A.A, Enick, R.M. and Panvelker, S.B (1984). An Analytical Extension Of The Dykstra-Parsons Vertical Stratification Discrete Solution To A Continuous, Real Time Basics, *Soc. Pet. Eng.J*. 24, 643-656.

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